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Miura, Kohtaroh; Lombardo, Maria Paola; Pallante, Elisabetta

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Thermodynamic Study for Conformal Phase in Large N_f Gauge Theory

Kohtaroh Miura*
INFN Laboratori Nazionali di Frascati, I-00044, Frascati (RM), Italy
E-mail: Kohtaroh.Miura@lnf.infn.it

Maria Paola Lombardo
INFN Laboratori Nazionali di Frascati, I-00044, Frascati (RM), Italy
Humboldt-Universität zu Berlin, Institut für Physik, D-12489 Berlin, Germany
E-mail: Mariapaola.Lombardo@lnf.infn.it

Elisabetta Pallante
Centre for Theoretical Physics, University of Groningen, 9747 AG, Netherlands
E-mail: e.pallante@rug.nl

We investigate the chiral phase transition at finite temperature (T) in colour SU$(N_c = 3)$ Quantum Chromodynamics (QCD) with six species of fermions ($N_f = 6$) in the fundamental representation [1]. The simulations have been performed by using lattice QCD with improved staggered fermions. The critical couplings $\beta_{\rm L}{}^{\rm c}$ for the chiral phase transition are observed for several temporal extensions N_t, and the two-loop asymptotic scaling of the dimensionless ratio $T_c/\Lambda_{\rm L}$ ($\Lambda_{\rm L}$ = Lattice Lambda-parameter) is found to be achieved for $N_t \geq 6$. Further, we collect $\beta_{\rm L}{}^{\rm c}$ at $N_f = 0$ (quenched), and $N_f = 4$ at a fixed $N_t = 6$ as well as $N_f = 8$ at $N_t = 6, 12$, the latter relying on our earlier study. The results are consistent with enhanced fermionic screening at larger N_f. The ratio $T_c/\Lambda_{\rm L}$ depends very mildly on N_f in the $N_f = 0-4$ region, begins increasing at $N_f = 6$, and significantly grows up at $N_f = 8$, as N_f reaches to the edge of the conformal window. We discuss the interrelation of the results with preconformal dynamics in the light of a functional renormalization group analysis.

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1. Introduction

Emergence of a conformal symmetry and a preconformal (walking) behavior in strongly flavored non-Abelian gauge theories has received much attention. Walking dynamics near the infrared fixed point has been advocated as a basis for strongly interacting mechanisms of electroweak symmetry breaking. Lattice Monte-Carlo simulations are expected to provide a solid theoretical base to understand the (pre-)conformal nature in the gauge theory.

A second zero of the two-loop beta-function of massless QCD with N_f flavours implies, at least perturbatively, the appearance of an infrared fixed point (IRFP) at $N_f \gtrsim 8.05$ [2] with the restoration of conformal symmetry before the loss of asymptotic freedom (LAF) at $N_f^{\mathrm{LAF}} = 16.5$. Conformality should emerge when the renormalized coupling at the would be IRFP is not strong enough to break chiral symmetry. This condition provides the lower bound N_f^c of a so called conformal window in the flavor space, and we find elaborated analytic predictions [3, 4]: for instance, the functional renormalization group method [5] suggests $N_f^c \sim 12$. Before the emergence of conformal symmetry, a qualitative change of dynamics is claimed at $N_f = 6$ based on instanton study [6].

Recent lattice studies[7] focused on the computation of the edge of the conformal window ${N_f}^c$ and the analysis of the conformal window itself, either with fundamental fermions [8 – 15]. or other representations [16]. Among the many interesting results with fundamental fermions, we single out the observation that QCD with three colours and eight flavours is still in the hadronic phase [9, 10], while $N_f = 12$ seems to be close to ${N_f}^c$, with some groups favouring conformality [8, 9, 11, 12], and others chiral symmetry breaking [14]. The onset of new strong dynamics at $N_f = 6$ has been implied via an enhancement of the ratio of chiral condensate to cubed pseudoscalar decay constant [17].

Using the thermal transition as a tool for investigating preconformal dynamics has been largely inspired by a renormalization group analysis [5]. The critical temperature for the chiral phase transition has been obtained as a function of N_f. Then the onset of the conformal window has been estimated by locating the vanishing critical temperature. The phase transition line is almost linear with N_f for small N_f, and clearly elucidates the universal critical behaviour at zero and non-zero temperature in the vicinity of N_f^c. Thus, it would be a promising direction to extend the knowledge of finite T lattice QCD to the larger N_f region, by using the FRG results as analytic guidance.

In this proceedings, we investigate the thermal chiral phase transition for $N_f = 6$ colour $\mathrm{SU}(N_c = 3)$ QCD by using lattice QCD Monte Carlo simulations with improved staggered fermions based on our recent study [1]. $N_f = 6$ is expected to be in the important regime as suggested by the results in Refs. [6, 8]. We also compute the critical couplings for $N_f = 0$ (quenched) and $N_f = 4$ at $N_t = 6$, and use the results from Ref. [10] for $N_f = 8$. Then we investigate N_f dependences of the chiral phase transition.

2. Simulation setups

Simulations have been performed in the same as in the study used for $N_f = 8$ in Ref. [10]: We have utilized the publicly available MILC code [18] with the use of an improved version of the staggered action, the Asqtad action, with a one-loop Symanzik [19, 20] and tadpole [22] improved gauge action. The tadpole factor u_0 is determined by performing zero temperature simulations on the 12^4 lattice, and used as an input for finite temperature simulations.

To generate configurations with mass degenerate dynamical flavours, we have used the rational hybrid Monte Carlo algorithm (RHMC) [21]. Simulations for $N_f = 6$ have been performed by using two pseudo-fermions, and subsets of trajectories for the chiral condensates and Polyakov loop have been compared with those obtained by using three pseudo-fermions with the same Monte Carlo time step $d\tau$ and total time length τ of a single trajectory. We have observed very good agreement between the two cases for both evolution and thermalization. We have monitored the Metropolis acceptance and reject ratio, and adjusted $\tau = 0.2 - 0.24$ and $d\tau = 0.008 - 0.016$ to realize the best performance.

Measured observables are the expectation values of the chiral condensate and Polyakov loop,

$$a^3\langle\bar{\psi}\psi\rangle = \frac{N_f}{4N_s^3 N_t}\left\langle \mathrm{Tr}\left[\mathrm{M}^{-1}\right]\right\rangle , \quad L = \frac{1}{N_c N_s^3}\sum_{\mathbf{x}} \mathrm{Re}\left\langle \mathrm{tr}_c \prod_{t=1}^{N_t} U_{4,t\mathbf{x}}\right\rangle , \tag{2.1}$$

where N_s (N_t) represents the number of lattice sites in the spatial (temporal) direction, $U_{4,t\mathbf{x}}$ is the temporal link variable, and tr_c denotes the trace in colour space. The output of this measurement is the critical coupling $\beta_{\mathrm{L}}{}^{\mathrm{c}}$ for the chiral phase transition.

3. Results

All results have been obtained for a fermion bare lattice mass $am = 0.02$. In the left panel of Figs. 1, the expectation values of the chiral condensate $a^3\langle\bar{\psi}\psi\rangle$ are displayed as a function of β_{L} for several N_t. It is found that different N_t give a different behaviour of $a^3\langle\bar{\psi}\psi\rangle$. The asymptotic scaling analysis below will confirm that it corresponds to a thermal chiral phase transition (or crossover) in the continuum limit.

All values of the critical lattice coupling $\beta_{\mathrm{L}}{}^{\mathrm{c}}$ are summarized in Table 1. For larger N_t, the signal for the chiral phase transition becomes less clear, hence we investigate the histogram of the chiral condensate: The histogram for $N_t = 8$ exhibits the double-peak structure at $\beta_{\mathrm{L}} = 5.2$, *i.e.*, the competition between chirally symmetric and broken vacua. The critical coupling can be estimated as $\beta_{\mathrm{L}}{}^{\mathrm{c}} = 5.225(25)$ for $N_t = 8$. For $N_t = 12$, we also observe the double-peak structure in the histogram of the chiral condensate around $\beta_{\mathrm{L}} = 5.45$.

These results can be analyzed and interpreted in terms of the two-loop asymptotic scaling. Let us consider the two-loop lattice beta function,

$$\beta(g) = -(b_0 g^3 + b_1 g^5) , \tag{3.1}$$

$$(b_0,\, b_1) = \left((11 - 2N_f/3)/(4\pi)^2,\ (102 - 38N_f/3)/(4\pi)^4\right) , \tag{3.2}$$

for fundamental fermions in colour SU(3). From Eq. (3.1), we obtain the well known two-loop asymptotic scaling,

$$\Lambda_{\mathrm{L}}\ a(\beta_{\mathrm{L}}) = \left(2N_c b_0/\beta_{\mathrm{L}}\right)^{-b_1/(2b_0^2)} \exp\left[-\beta_{\mathrm{L}}/(4N_c b_0)\right] . \tag{3.3}$$

Here, Λ_{L} is the so-called lattice Lambda-parameter, and $\beta_L = 2N_c/g^2$, with $g = \sqrt{2N_c/10}\cdot g_{\mathrm{L}}$. This definition effectively takes account of the improvement of the staggered lattice action when comparing to the asymptotic scaling law, see Ref. [10]. We insert Λ_{L} to the definition of temperature

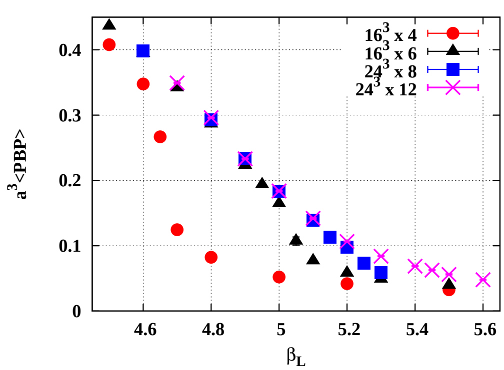
$T \equiv [a(\beta_{\mathrm{L}})N_t]^{-1}$,

$$N_t^{-1} = (T_c/\Lambda_{\mathrm{L}}) \times \left(\Lambda_{\mathrm{L}}\ a(\beta_{\mathrm{L}}{}^{\mathrm{c}})\right) , \tag{3.4}$$

and extract the physical quantity T_c/Λ_{L} by substituting the simulation outputs $\beta_{\mathrm{L}}{}^{\mathrm{c}}$ for Eq. (3.4). This ratio must be unique as long as the asymptotic scaling Eq. (3.3) is verified for a given $\beta_{\mathrm{L}}{}^{\mathrm{c}}$.

In the right panel of Fig. 1, the slope of the line connecting the origin and the data points corresponds to T_c/Λ_{L}. The $N_t = 6,\ 8$, and 12 points have a common slope to a very good approximation, while the $N_t = 4$ result falls on a smaller slope. The latter is interpreted as a scaling violation effect due to the use of a too small N_t. The existence of a common T_c/Λ_{L} for $N_t \geq 6$ indicates that the data are consistent with the two-loop asymptotic scaling Eq. (3.3), confirms the thermal nature of the transition and that $N_f = 6$ is outside the conformal window, as expected from a previous $N_f = 8$ study [10]. A linear fit provides $T_c/\Lambda_{\mathrm{L}} = 1.02(12) \times 10^3$, which can be interpreted as the value in the continuum limit for $N_f = 6$ QCD.

In order to have a more complete overview, we have performed simulations for the theory with $N_f = 0$ (quenched) and $N_f = 4$, only at $N_t = 6$. These theories are of course very well investigated, however we have not found in the literature results for the same action as ours. We note that in a previous lattice study with improved staggered fermions [23], asymptotic scaling was observed for $N_t \geq 6$ for $0 \leq N_f \leq 4$. Table 1 shows a summary of our results for the critical coupling $\beta_{\mathrm{L}}{}^{\mathrm{c}}$ of the chiral phase transition at finite temperature for $N_f = 0,\ 4,\ 6$, and 8 - the latter from Ref. [10].



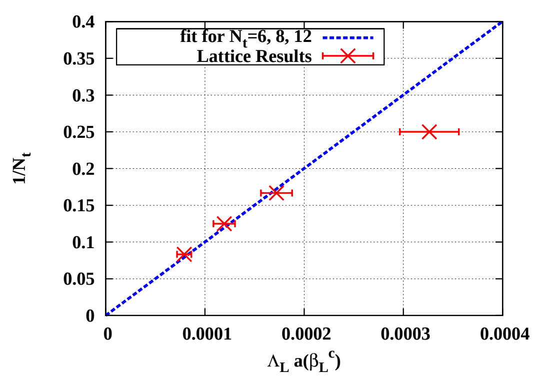


Figure 1: Left: The chiral condensate $a^3\langle\bar{\psi}\psi\rangle$ for $N_f = 6$ and $am = 0.02$ in lattice units, as a function of β_{L}, for $N_t = 4,\ 6,\ 8$, and 12. Error-bars are smaller than symbols. Right: The thermal scaling behaviour of the critical lattice coupling $\beta_{\mathrm{L}}{}^{\mathrm{c}}$. Data points for $\Lambda_{\mathrm{L}}\ a(\beta_{\mathrm{L}}{}^{\mathrm{c}})$ at a given $1/N_t$ are obtained by using $\beta_{\mathrm{L}}{}^{\mathrm{c}}$ from Table 1 as input for extracting $\Lambda_{\mathrm{L}}\ a(\beta_{\mathrm{L}}{}^{\mathrm{c}})$ in the two-loop expression Eq. (3.3). The dashed line is a linear fit with zero intercept to the data with $N_t > 4$.

In the left panel of Fig. 2, we display the critical values of the lattice coupling $g_c = \sqrt{2N_c/\beta_{\mathrm{L}}{}^{\mathrm{c}}}$ from Table 1 in the Miransky-Yamawaki phase diagram. Consider the $N_t = 6$ results: it is expected that an increasing number of flavours favors chiral symmetry restoration. Indeed, we find that, on a fixed lattice, the critical coupling increases with N_f in agreement with early studies and naive reasoning. The precise dependence of the critical coupling on N_f at fixed N_t is not known. It is, however, amusing to note that the results seem to be smoothly connected by an almost straight line: the brown line in the plot is a linear fit to the data. Comparing the trend for $N_f = 6$ to the one for

$N_f = 8$ for varying N_t, one can infer a decreasing in magnitude (and small) step scaling function, hence a walking behaviour. Further study is needed at larger N_f, and by using the same action used for $N_f = 0-8$, to confirm or disprove it.

Table 1: Summary of the critical lattice couplings $\beta_{\rm L}{}^{\rm c}$ for the theories with $N_f = 0,\ 4,\ 6,\ 8$, $am = 0.02$ and varying $N_t = 4,\ 6,\ 8,\ 12$. All results are obtained using the same lattice action.

$N_f \backslash N_t$	4	6	8	12
0	-	7.88 ± 0.05	-	-
4	-	5.89 ± 0.03	-	
6	4.675 ± 0.025	5.025 ± 0.025	5.225 ± 0.025	5.45 ± 0.05
8	-	4.1125 ± 0.0125	-	4.34 ± 0.04

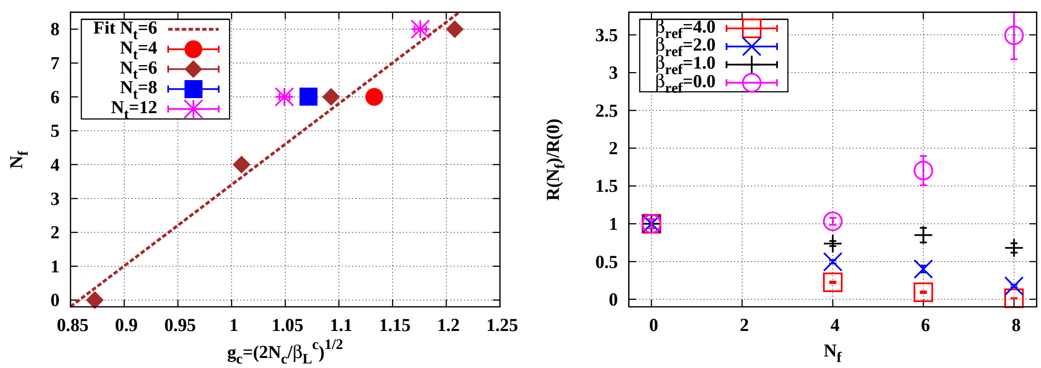


Figure 2: Left: Critical values of the lattice coupling $g_c = \sqrt{2N_c/\beta_{\rm L}{}^{\rm c}}$ for theories with $N_f = 0,\ 4,\ 6,\ 8$ and for several values of N_t in the Miransky-Yamawaki phase diagram. The dashed (brown) line is a linear fit to the $N_t = 6$ results. Right: The N_f dependence of $R(N_f)/R(0)$ for several finite fixed $\beta_{\rm L}{}^{\rm ref}$. Here, $R(N_f) \equiv (T_c/\Lambda_{\rm ref})(N_f)$.

Next, we study the N_f dependence of the ratio $T_c/\Lambda_{\rm L}$ and related quantities. In addition to the scale $\Lambda_{\rm L}$, we introduce more UV reference energy scale $\Lambda_{\rm ref}$, which is associated with a reference coupling $\beta_{\rm L}{}^{\rm ref}$. Then Eq. (3.3) is generalized as

$$\Lambda_{\rm ref}(\beta_{\rm L}{}^{\rm ref})\, a(\beta_{\rm L}) = \left(\frac{b_1}{b_0^2}\ \frac{\beta_{\rm L} + 2N_c b_1/b_0}{\beta_{\rm L}{}^{\rm ref} + 2N_c b_1/b_0}\right)^{b_1/(2b_0^2)} \exp\left[-\frac{\beta_{\rm L} - \beta_{\rm L}{}^{\rm ref}}{4N_c b_0}\right] . \tag{3.5}$$

At leading order of perturbation theory $b_1 \to 0$, we find $\Lambda_{\rm ref}/\Lambda_{\rm L} = \exp[\beta_{\rm L}{}^{\rm ref}/(4N_c b_0)]$. This equation would be analogous of the ratio $\Lambda_{\rm L}/\Lambda_{\rm MS}$ derived in [24] for Wilson fermions up to a further linear dependence on N_f in the numerator of the exponent. In a nutshell, the difference originates from the fact that we are fixing a bare reference coupling $\beta_{\rm L}{}^{\rm ref}$, which will be specified later. Notice that by construction $\Lambda_{\rm ref}$ reproduces the lattice Lambda-parameter $\Lambda_{\rm L}$ in the limit $\Lambda_{\rm ref}(\beta_{\rm L}{}^{\rm ref} \to 0) = \Lambda_{\rm L}\left(1 + \mathscr{O}(1/\beta_{\rm L}{}^{\rm c})\right)$.

Let us consider first $R(N_f)|_{\beta_{\rm L}{}^{\rm ref}=0.0} = T_c/\Lambda_{\rm L}$. The values of $T_c/\Lambda_{\rm L}$ are found to be 600 ± 34, 620 ± 28, 1023 ± 117, and 2098 ± 191 for $N_f = 0$, 4, 6, and 8, respectively, and represented as circles in the right panel of Fig. 2 (the vertical axis is normalized by $R(0) = (T_c/\Lambda_{\rm L})(N_f = 0)$ for each $\beta_{\rm L}{}^{\rm ref}$). The ratio does not show a significant N_f dependence in the region $0 \leq N_f \leq 4$, it starts increasing at $N_f = 6$, and undergoes a rapid rise around $N_f = 8$. The nearly constant nature of $T_c/\Lambda_{\rm L}$ in the region $N_f \leq 4$ indicates that the role of such energy scale is not significantly changed by the variation of N_f (see [25] for a detailed discussion of this point.) In turn, the increase of $T_c/\Lambda_{\rm L}$ in the region $N_f \geq 6$ might well imply that the chiral dynamics becomes different from the one for $N_f \leq 4$. Indeed, a recent lattice study [17] indicates that $N_f = 6$ is close to the threshold for preconformal dynamics.

We now consider $T_c/\Lambda_{\rm ref}$ with finite $\beta_{\rm L}{}^{\rm ref}$. The N_f dependence of the ratio $R(N_f) \equiv (T_c/\Lambda_{\rm ref})(N_f)$ is shown for several $\beta_{\rm L}{}^{\rm ref}$ in the right panel of Fig. 2, (with normalization by $R(0) = (T_c/\Lambda_{\rm ref})(N_f = 0)$ for each $\beta_{\rm L}{}^{\rm ref}$). $T_c/\Lambda_{\rm ref}$ is now a decreasing function of N_f for a larger $\beta_{\rm L}{}^{\rm ref}$. The $\Lambda_{\rm ref}$ associated with a $\beta_{\rm L}{}^{\rm ref} \gg \beta_* = 2N_c/g^2_{\rm IRFP}$ would be less sensitive to the IR or chiral dynamics. Assuming $N_f^c \simeq 12$, the two-loop beta-function leads to $\beta_* = -2N_c b_1/b_0 \simeq 0.63$. The decreasing nature of $(T_c/\Lambda_{\rm ref})(N_f)$ is found to start around $\beta_{\rm L}{}^{\rm ref} = 1.0 \gtrsim \beta_*$. Thus, the use of a UV reference scale leads to the decreasing $(T_c/\Lambda_{\rm ref})(N_f)$. This trend is consistent with the FRG study [5], where the decreasing $T_c(N_f)$ has been obtained by using the τ-lepton mass m_τ as a common UV reference scale with a common coupling $\alpha_s(m_\tau)$. We note that we have constrained our analyses $\beta_{\rm L}{}^{\rm ref} < \beta_{\rm UV} = \beta_{\rm L}{}^{\rm c}(N_f) \leq 4.1125\pm 0.0125$.

With the use of a UV reference scale, we should observe the predicted critical behavior [5],

$$T_c(N_f) = K|N_f - N_f^c|^{-1/\theta} \ . \tag{3.6}$$

By choosing the critical exponent θ in the range predicted by FRG: $1.1 < 1/|\theta| < 2.5$, our data are consistent with the values $N_f^c = 9(1)$ for $\beta_{\rm L}{}^{\rm ref} = 4.0$ and $N_f^c = 11(2)$ for $\beta_{\rm L}{}^{\rm ref} = 2$. We plan to extend and refine this analysis in the future, and here we only notice a reasonable qualitative behaviour.

4. Summary

We have studied the chiral phase transition at finite T for colour $SU(3)$ QCD with $N_f = 6$ by using lattice QCD Monte-Carlo simulations with improved staggered fermions [1]. We have determined the critical lattice coupling $\beta_{\rm L}{}^{\rm c}$ for several lattice temporal extensions N_t, and extracted the dimensionless ratio $T_c/\Lambda_{\rm L}$ ($\Lambda_{\rm L}$ =Lattice Lambda-parameter) by using two-loop asymptotic scaling. The analogous result for $N_f = 8$ has been extracted from Ref. [10]. $T_c/\Lambda_{\rm L}$ for $N_f = 0$ and $N_f = 4$ has been measured at fixed $N_t = 6$, barring asymptotic scaling violations. Then we have discussed the N_f dependence of the ratios $T_c/\Lambda_{\rm L}$ and $T_c/\Lambda_{\rm ref}$, where $\Lambda_{\rm ref}$ is a UV reference energy scale, related to $\Lambda_{\rm L}$ via $\Lambda_{\rm ref}/\Lambda_{\rm L} \simeq \exp[\beta_{\rm L}{}^{\rm ref}/(4N_c b_0)]$. We have observed that $T_c/\Lambda_{\rm L}$ shows an increase in the region $N_f = 6-8$, while it is approximately constant in the region $N_f \leq 4$. We have discussed this qualitative change for $N_f \geq 6$ and a possible relation with a preconformal phase. The ratio $T_c/\Lambda_{\rm ref}$ is a decreasing function of N_f. This behaviour is consistent with the result obtained in the functional renormalization group analysis [5]. Next steps of the current project involve a scale setting at zero temperature by measuring a common UV observable.

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